$$f=\frac{e^{i5x}-e^{ix}+3e^{i3x}-3e^{-ix}+3e^{ix}-3e^{-i3x}+e^{-ix}-e^{-i5x}}{16i}$$

$$f=\frac{(e^{i5x}-e^{-i5x})+2(e^{ix}-e^{-ix})+3(e^{i3x}-e^{-i3x})}{16i}$$

$$f=\frac{2i\sin 5x+4i\sin x+6i\sin 3x}{16i}=\frac{1}{8}\sin 5x+\frac{1}{4}\sin x+\frac{3}{8}\sin 3x$$

2°/ $g=\cos^2 2x\sin 3x=\left(\frac{e^{i2x}+e^{-i2x}}{2}\right)^2\left(\frac{e^{i3x}-e^{-i3x}}{2i}\right)$

$$=\frac{(e^{i4x}+2+e^{-i4x})(e^{i3x}-e^{-i3x})}{8i}$$

$$=\frac{e^{i7x}-e^{ix}+2e^{i3x}-2e^{-i3x}+e^{-ix}-e^{-i7x}}{8i}$$

$$=\frac{(e^{i7x}-e^{-i7x})-(e^{ix}-e^{-ix})+2(e^{i3x}-e^{-i3x})}{8i}$$

$$=\frac{2i\sin 7x+4i\sin 3x-2i\sin x}{8i}=\frac{1}{4}\sin 7x+\frac{1}{2}\sin 3x-\frac{1}{4}\sin x$$

3°/

$$h=\cos^4 x\sin x=\left(\frac{e^{ix}+e^{-ix}}{2}\right)^4\left(\frac{e^{ix}-e^{-ix}}{2i}\right)=\frac{\left(e^{ix}+e^{-ix}\right)^4\left(e^{ix}-e^{-ix}\right)}{32i}$$

Or $\left(e^{ix}+e^{-ix}\right)^4=e^{i4x}+4e^{i2x}+6+4e^{-i2x}+e^{-i4x}$

d'où $h=\frac{(e^{i4x}+4e^{i2x}+6+4e^{-i2x}+e^{-i4x})(e^{ix}-e^{-ix})}{32i}$

$$=\frac{e^{i5x}-e^{i3x}+4e^{i3x}+4e^{ix}+6e^{ix}-6e^{-ix}+4e^{-ix}-4e^{-i3x}+e^{-i3x}-e^{-i5x}}{32i}$$

$$=\frac{(e^{i5x}-e^{-i5x})+3(e^{i3x}-e^{-i3x})+10(e^{ix}-e^{-ix})}{32i}$$

$$=\frac{2i\sin 5x+6i\sin 3x+20i\sin x}{32i}$$

Conclusion : $h=\frac{1}{16}\sin 5x+\frac{3}{16}\sin 3x+\frac{5}{8}\sin x$